

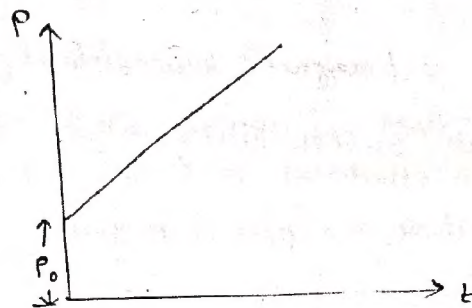
## Super Conductivity :-

→ Conductivity of a material  $\propto \frac{1}{\text{Resistivity}(\rho)}$

The ideal conductor; i.e., conductivity ( $\alpha$ ) is infinite and resistivity is zero; is called Super Conductor.

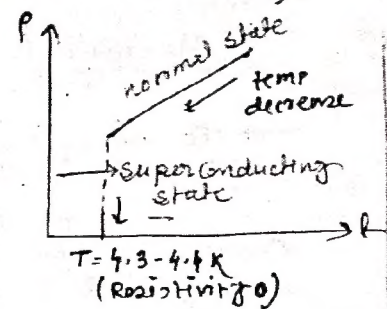
→ For any conductor if we decrease the temperature; resistivity decreases; conductivity increases.

$$\rho = \rho_0 (1 + \alpha t) \rightarrow \text{Theoretically}$$



In 1960 → Kamerlingh  
descovered  
Liquid Helium.  
(4.2 K)

→ For Hg (Room temp. liquid)



→ 1911 first time superconductivity was observed.  
 $T = 4.3-4.4 \text{ K}$ ; this temp. is called transition temperature.

[room temp. super conductor is not possible.]  
(145 K) → " " "

$\rho \rightarrow 0$  at  $T = 4.3-4.4 \text{ K}$   
Experimental graph

→ Property of Semi-Conductor :-

- (i) Resistivity of superconductor is zero.
- (ii) A superconductor is a perfect diamagnet.

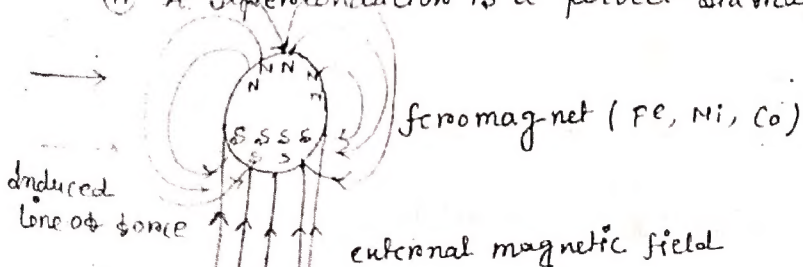


fig: for ferromagnetic material

So total magnetic field,  $\vec{B}_{\text{net}} = \text{induced magnetic field} + \text{applied magnetic field}$

$$\vec{B}_{\text{net}} = \mu_0 \vec{M} + \mu_0 \vec{H}_{\text{app}}$$

$$\vec{B}_{\text{net}} = \mu_0 (\vec{M} + \vec{H}_{\text{app}}) ; \frac{\vec{B}}{\vec{H}} = \chi$$

MRI used

→ Super Conduc

Room temp →

super good

Conduction → GOLD

SILVER

platinum

super conducting

exist at 4.2 K

Lead → super

conducting

exist at 4.2 K

but bad

conductor

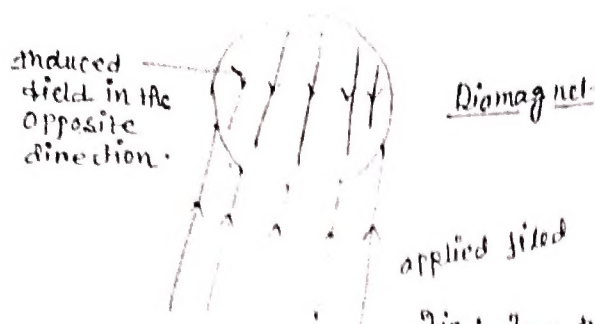


Fig: Non perfect diamagnet

(Superconductor is perfect diamagnet)

$$\vec{B}_{\text{total}} = 0$$

$$\vec{M} + \vec{H}_{\text{app}} = 0$$

or,  $\frac{\vec{M}}{H} = -1 = \chi$  (magnetic susceptibility)

For perfect diamagnetic susceptibility will be  $-1$ . This phenomenon is known as Meissner effect.

A Superconductor behaves as a perfect magnet.

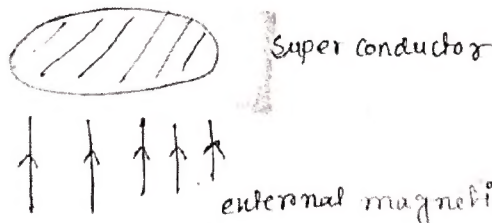
Magnetic levitation ( $\text{Nb}_3\text{Sn} \rightarrow \text{superconductor}$ )

→ To use superconductor in our daily life; a superconductor is required with high transition temp. ( $T_c$ ).

The highest possible transition temperature for the superconducting material is  $\text{HgBaCaCuO}$  (Mercury Barium Calcium Copper oxide) with  $T_c = 135 \text{ K}$  ( $-138^\circ\text{C}$ ).

$\text{YBaCuO}$  (Yttrium Barium Copper oxide);  $T_c = 90 \text{ K}$

→ Superconductivity of a material can be destroyed by using external magnetic field.



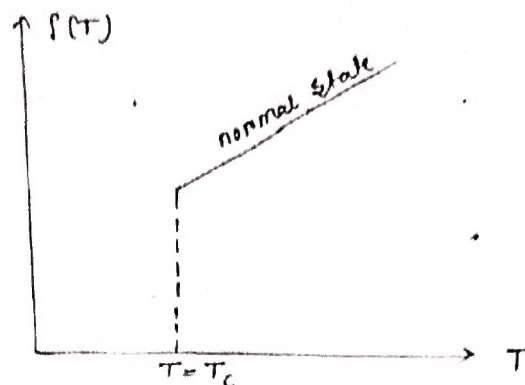
[The magnetic field at which a superconductor becomes a normal conductor; is called critical magnetic field ( $H_c$ )]

→ [When current flows through a superconductor; this current produces a magnetic field. When the current is sufficient to produce critical magnetic field for the superconductor, conducting state will be destroyed. This limit of current is called critical current and density is called critical current density ( $J_c$ )] Corresponding current



Critical current density ( $J_c$ )

for a good <sup>super</sup>conductor  $T_c$ ,  $H_c$ ,  $J_c$  should be high



1)  $T > T_c$  ;  $H_c = 0$   
 $T < T_c$  ;  $H_c \neq 0$

from the above two relation, we can say that the  $H_c$  depends on temperature. We get an empirical relation between  $H_c$  and  $T$  as

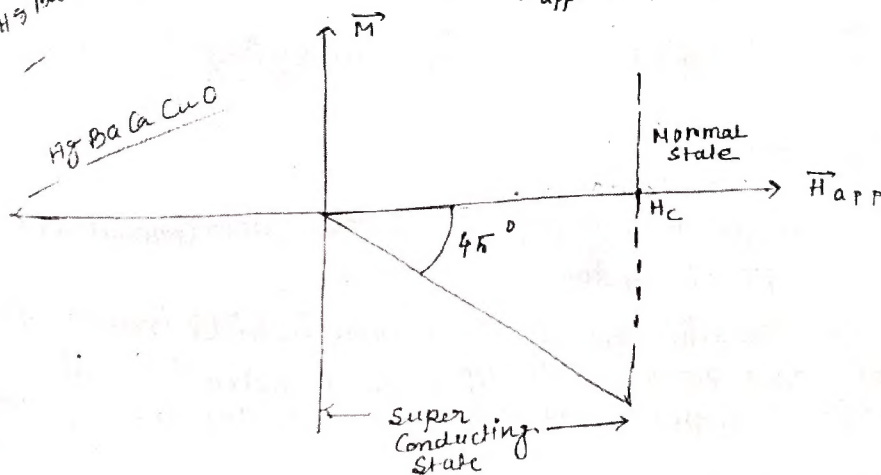
$$H_c(T) = H_c(0) \left[ 1 - \left( \frac{T}{T_c} \right)^2 \right] ; T < T_c$$

$H_c(0) \rightarrow 0$  Kelvin temp. magnetic field

$H_c(0)$  = critical magnetic field at 0 Kelvin

$$J_c(T) = J_c(0) \left[ 1 - \left( \frac{T}{T_c} \right)^2 \right] ; T < T_c$$

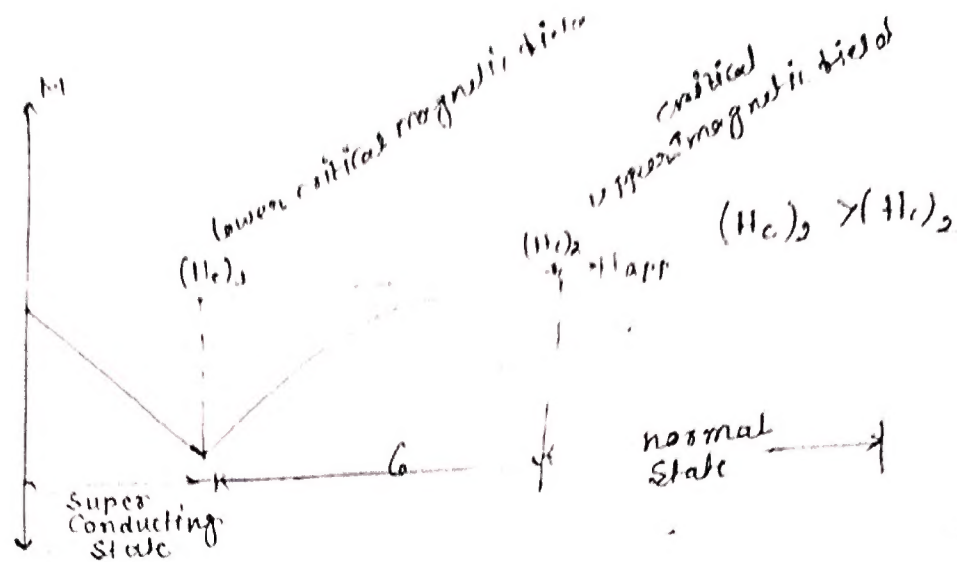
$H_c$  calculated from Meissner effect ;  $\frac{\vec{M}}{H_{app}} = \chi = -1$



The Superconductor which follows the Meissner effect is called Type-I Superconductor.

Type-1, example : Nb ; Hg ; Pb

They are some <sup>neobium</sup> Superconductors which do not follow the above graph; they are called Type-2 Superconductor.



As  $(H_c)_2$  is greater we can pass more current through type-II superconductor as compare to type-I superconductor.

Type-2 Example,  
 $Nb_3Sn / Nb_3Al \rightarrow (H_c)_2$  is 44 Tesla | more current  $J_c$   $\propto H_c$   
 at  $T \approx 0K$

→ In MRI, high magnetic field is used; to produce such high magnetic field by using normal ferromagnetic material is impossible.

$Nb_3Al \rightarrow (H_c)_2$  44 Tesla

$T_c = 20K / 21K$

↓  
 use in MRI → (magnetic resonance imaging)

London's equation: =

The electrodynamic equations of the superconductor was given by Heinz and Fritz London.

As discussed in Meissner effect the magnetic field inside the superconductor is not drop zero suddenly. The penetration of magnetic field through a superconductor is explained by using London's equation.

London's eqn considered that there are two types of electron inside a superconductor;

- Superconducting electron
- normal electron

### 1st London eqn :

We Considered,  $n_s$  is number density of superconducting electron.

$V_s$  = velocity of superconducting electron.

force on superconducting electron due to applied electric field,  
 $= -eE$ ;

$E$  = applied electric field

$e$  = charge.

$$\therefore m \frac{dV_s}{dt} = -eE \quad \text{--- (1)}$$

current density;

$$J = -n_s V_s e, \text{ where } V_s = \text{velocity of superconducting electron,}$$

$$\text{or, } V_s = -\frac{J_s}{en_s}$$

$$\frac{dV_s}{dt} = -\frac{1}{en_s} \frac{dJ_s}{dt} \quad \text{--- (11)}$$

$J_s$  = superconducting current density

replacing eqn (11) in (1)  $\Rightarrow$

$$-m \frac{1}{en_s} \frac{dJ_s}{dt} = -eE \quad ; \quad \frac{dJ_s}{dt} = \frac{e^2 n_s E}{m}$$

This is known as London first equation.

Q. How entropy  $ch$  is related in superconducting transition.  
theoretically  
--- Mark-2

2nd London equation:

$$\frac{d\vec{J}_s}{dt} = \frac{e^2 n_s}{m} \vec{E}$$

1st London eqn — (1)

Taking curl on the both side of eqn (1)

$$\vec{\nabla} \times \left( \frac{d\vec{J}_s}{dt} \right) = \frac{e^2 n_s}{m} (\vec{\nabla} \times \vec{E}) \quad \text{--- (II)}$$

from Maxwell eqn,

$$\vec{\nabla} \times \vec{E} = - \frac{\partial \vec{B}}{\partial t}$$

$$\text{From (II)} \quad \vec{\nabla} \times \left( \frac{d\vec{J}_s}{dt} \right) = - \frac{e^2 n_s}{m} \frac{\partial \vec{B}}{\partial t}$$

$$\Rightarrow \frac{\partial}{\partial t} \{ \vec{\nabla} \times \vec{J}_s \} = - \frac{e^2 n_s}{m} \frac{\partial \vec{B}}{\partial t}$$

$$\frac{\partial}{\partial t} \{ \vec{\nabla} \times \vec{J}_s \} = - \frac{\partial}{\partial t} \left\{ \frac{e^2 n_s}{m} \vec{B} \right\} \quad \text{--- (III)}$$

$e$  = charge of electron  
 $n_s$  = number of superconducting  $e^-$  per unit volume  
 $m$  = mass of  $e^-$

$$\vec{\nabla} = \hat{i} \frac{\partial}{\partial x} + \hat{j} \frac{\partial}{\partial y} + \hat{k} \frac{\partial}{\partial z}$$

$\vec{\nabla}$ , Delta operator independent of time.

$$\begin{aligned} \frac{\partial}{\partial t} (\vec{\nabla} \times \vec{J}) &= \frac{\partial}{\partial t} (\vec{\nabla}) \times \vec{J} + \vec{\nabla} \times \frac{\partial \vec{J}}{\partial t} \\ &= \vec{\nabla} \times \frac{\partial \vec{J}}{\partial t} \end{aligned}$$

Integrating both side of eqn (III) w.r. to time ( $t$ )

$$\int \frac{\partial}{\partial t} (\vec{\nabla} \times \vec{J}_s) dt = - \int \frac{\partial}{\partial t} \left( \frac{e^2 n_s}{m} \vec{B} \right) dt$$

$$\boxed{\vec{\nabla} \times \vec{J}_s = - \frac{e^2 n_s}{m} \vec{B}}$$

This is known as London 2nd eqn of Superconductivity.

Again ;  $\vec{B} = \vec{\nabla} \times \vec{A}$  ;  $\vec{A}$  = Vector potential

$$\vec{\nabla} \times \vec{J}_s = - \frac{e^2 n_s}{m} \vec{\nabla} \times \vec{A}$$

$$\Rightarrow \vec{\nabla} \times \vec{J}_s = \vec{\nabla} \times \left( - \frac{e^2 n_s}{m} \vec{A} \right)$$

$$\boxed{\vec{J}_s = - \frac{e^2 n_s}{m} \vec{A}}$$

From Ampere's law of magnetism ;

$$\text{using Stokes theorem, } \oint \vec{B} \cdot d\vec{r} = \mu_0 I$$

$$\Rightarrow \vec{\nabla} \times \vec{B} = \mu_0 \vec{J}_s \quad \text{--- (i)}$$

$I_s$  = Superconducting current  $= \iint \vec{J}_s \cdot d\vec{s}$

$\vec{J}_s$  = Superconducting current density

Taking curl on the both side of eqn (i)

$$\vec{\nabla} \times (\vec{\nabla} \times \vec{B}) = \mu_0 (\vec{\nabla} \times \vec{J}_s)$$



$$\vec{\nabla} \cdot (\vec{\nabla} \cdot \vec{B}) - \vec{\nabla} \cdot (\vec{\nabla} \times \vec{B}) = \mu_0 \left\{ \vec{\nabla} \cdot \vec{J} - \frac{n_s e^2 \vec{A}}{m} \right\} \quad \begin{matrix} \vec{\nabla} \cdot \vec{B} = 0 \\ \vec{\nabla} \times \vec{A} = \vec{B} \end{matrix}$$

$$\Rightarrow -\vec{\nabla}^2 \vec{B} = -\frac{\mu_0 n_s e^2}{m} (\vec{\nabla} \times \vec{A})$$

$$\Rightarrow -\vec{\nabla}^2 \vec{B} = -\frac{\mu_0 n_s e^2}{m} \vec{B}$$

$$\Rightarrow \left[ \vec{\nabla}^2 \vec{B} = \frac{\mu_0 n_s e^2}{m} \vec{B} \right]$$

for one dimension;

$$\therefore \frac{\partial^2 \vec{B}}{\partial x^2} = \frac{\mu_0 n_s e^2}{m} \vec{B} \quad ; \quad \frac{m}{\mu_0 n_s e^2} = \begin{cases} \frac{m}{\frac{e}{I}} \\ \frac{m}{\frac{e}{L^3} e^2} \\ \frac{L^3 m}{I^2} \end{cases}$$

$$\frac{m}{\mu_0 n_s e^2} = [L^2]$$

$$[n_s] = [L^{-3}]$$

$$\Rightarrow c = \frac{1}{\sqrt{\mu_0 \epsilon_0}}$$

$$\Rightarrow \mu_0 \epsilon_0 = \frac{1}{c^2}$$

$$c = \frac{\phi^2}{\partial v}$$

$$[\epsilon_0] = \left[ \frac{(IT)^2}{ML^2T^{-2}} \right]$$

$$\Rightarrow \mu_0 \left[ \frac{I^2 T^2}{ML^3 T^{-2}} \right] = \frac{1}{[LT^{-1}]^2}$$

$$\Rightarrow \mu_0 = \left[ \frac{ML^3 T^{-2}}{I^2 T^2 L^2 T^2} \right]$$

$$= \left[ \frac{MLT^2}{I^2} \right]$$

$$[e] = [IT]$$

$$\lambda_L^2 = \left( \frac{m}{\mu_0 n_s e^2} \right)$$

$$\lambda_L = \text{penetration depth} = [L]$$

$$\Rightarrow \frac{\partial^2 \vec{B}}{\partial x^2} = \frac{1}{\lambda_L^2} \vec{B}$$

$$\Rightarrow \left( \frac{\partial^2 \vec{B}}{\partial x^2} - \frac{1}{\lambda_L^2} \vec{B} \right) = 0$$

this is a second order homogeneous differential eqn,

$$m^2 = \frac{1}{\lambda_L^2}$$

$$\Rightarrow m = \frac{1}{\lambda_L}$$

$$\Rightarrow m = \pm \frac{1}{\lambda_L}$$

$$\vec{B} = c_1 e^{-x/\lambda_L} + c_2 e^{x/\lambda_L}$$



Surface induction  
more  
less

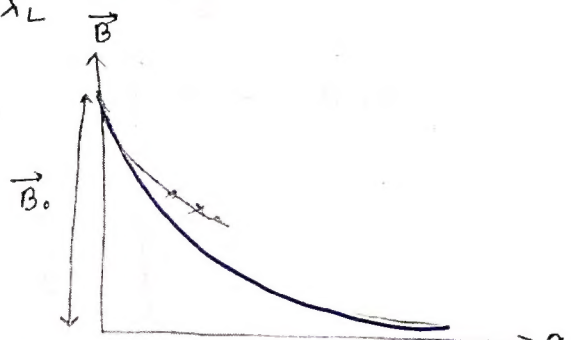
$\vec{B}$  = magnetic lines of force through superconductor.

The and term indicate magnetic induction <sup>increase in</sup> with distance from the surface of superconductor which is not true,

$$\vec{B} = c_1 e^{-x/\lambda_L}$$

Let at  $x=0$  (at the surface superconductor) ;  $\vec{B} = \vec{B}_0$  ;  $c_1 = \vec{B}_0$   
(magnetic at the surface)

$$\vec{B} = \vec{B}_0 e^{-x/\lambda_L}$$



The distance at which the magnetic field is become  $\frac{1}{e}$  times of magnetic field at the surface of superconductor is called London penetration depth ( $\lambda_L$ )

The penetration depth is directly depend on temperature ( $T$ ).  
with increase in temp from transition <sup>temp</sup> ( $T_c$ )  $\lambda_L$  will increase.

The empirical relation between  $\lambda_L$  and  $T$ ;  
experimental

$$\lambda_L = \lambda_0 \left[ 1 - \frac{T}{T_c} \right]^{1/2}$$

;  $T_c$  = transition temp.

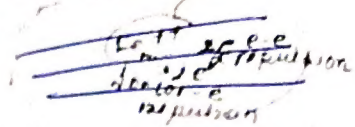
at,  $T = T_c$ ,  $\lambda_L = 0$

$\lambda_0$  = penetration depth at 0 Kelvin

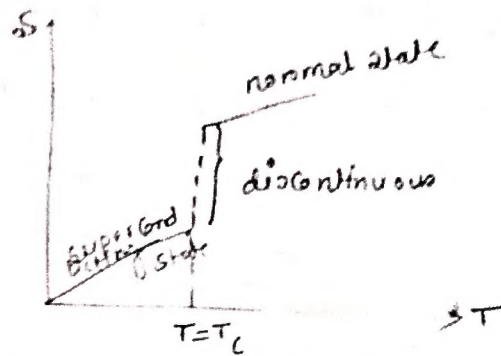


Energy of Superconducting state and normal state :-

At superconducting state the resistance of superconductor is almost zero and in normal state due to electron-electron collision and electron-lattice point collision there is a significant resistance.

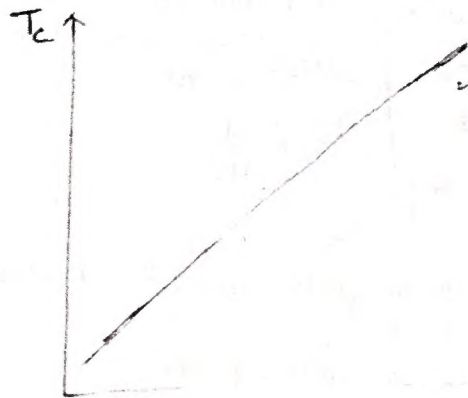


So, Superconducting state is more ordered system as compare normal state. Therefore, entropy in superconducting is less compare to normal state.



Quantum theory of Superconductivity :-

→ In 1950 E. Maxwell and Reynold ; experimentally discover relation between transition temperature ( $T_c$ ) and isotopic mass of superconductor by using isotopes of Hg.



$\sqrt{\frac{1}{M}}$  ; M = mass of isotope

$$T_c \propto \sqrt{\frac{1}{M}} \rightarrow \text{Experimental result}$$

Hg<sup>200</sup>, Hg<sup>202</sup>, Hg<sup>201</sup>, Hg

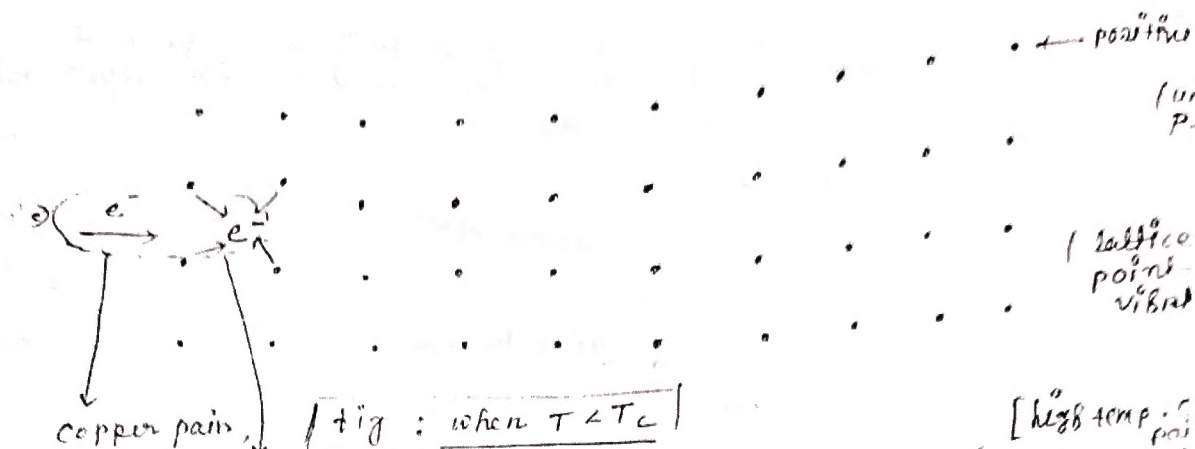
To explain this relation ; John Bardeen , Leon Cooper , John Schrieffer proposed an important theory in 1972 known as BCS theory.

Copper pairs :  
in superconductivity ,  
phonon wave

## B.C.S Theory

### Copper pair

In metal there is large number of electron and positive ions cores.



when  $T < T_c$   
there is no collision (Resistance = 0)

phonon wave  
amplitude

Energy of phonon wave is called phonon energy / quanta lattice vibration.

Lattice point heavy vibration  
← Hg 200  
Pb 208  
vibration 205

The vibration of phonon wave depends mass of each lattice point.

$$T_c \propto \frac{1}{\sqrt{M}}$$

[high temp. pair]  
[low temp. pair]  
Copper pair

$$\text{Spin copper pair} = \frac{1}{2} - 0$$

Q. find the minimum magnetic field needed for the Zeeman effect to be

Q. Calculate the value of London penetration depth  $\lambda_L$  at 0 K for lead whose density is  $11.3 \times 10^3 \text{ kg/m}^3$  and the atomic weight is 207.19. in  $T_c$  is 7.22 K.

$$\lambda_L = \frac{m}{\mu_0 n_s e^2}$$

$$n_s = 11.3 \times 10^3 \text{ kg/m}^3$$

$$M = 207.19 \text{ amu} = 207.19 \times 10^{-27} \text{ kg}$$

$$T_c = 7.22 \text{ K}$$

$$\lambda_L = \sqrt{\frac{207.19 \times 1.6 \times 10^{-27}}{4\pi \times 10^{-7} \times 11.3 \times 10^3 \times (1.6 \times 10^{-19})^2}}$$

$$= \sqrt{\frac{207.19}{4\pi \times 11.3 \times 1.6} \times 10^{-27+7-3+38}} \times 10^{15}$$

$$= 0.912 \times 10^7 \times 10^7 = 3.08 \times 10^7 \text{ m}$$

Find the minimum magnetic field needed for the Zeeman effect to be resolved in a spectral line of 400 nm wavelength when a spectrometer resolution is 0.010 nm is used.

Soln:  $\lambda = 400 \text{ nm}$  (Wavelength of original spectra)  $= 400 \times 10^{-9} \text{ m}$   
 $\Delta\lambda = 0.010 \text{ nm}$  (Zeeman shift)  $= 0.01 \times 10^{-9}$

Zeeman shift in terms of frequency,

$$\Delta\nu = \frac{\mu_B B}{h} \quad \text{--- (1)}$$

we know that;  $v = \frac{c}{\lambda}$

$$\Rightarrow \Delta\nu = - \frac{c}{\lambda^2} \Delta\lambda$$

$$|\Delta\nu| = \left| \frac{c}{\lambda^2} \right| \Delta\lambda \quad \text{--- (2)}$$

from eqn (1) and (2)

$$\frac{c}{\lambda^2} \Delta\lambda = \frac{\mu_B B}{h}$$

$$\therefore B = \frac{hc \Delta\lambda}{\lambda^2 \mu_B}$$

$$\mu_B = 9.27 \times 10^{-24} \text{ J/T}$$

$$h = 6.626 \times 10^{-34} \text{ J/Sec}$$

$$c = 3 \times 10^8 \text{ m/sec}$$

$$= \frac{6.626 \times 10^{-34} \times 3 \times 10^8 \times 0.01 \times 10^{-9}}{400 \times 10^{-9} \times 400 \times 10^{-9} \times 9.27 \times 10^{-24}}$$

$$= \frac{0.19878}{14.832} \times 10^{-3} \quad \frac{10^{-43}}{10^{-43}}$$

$$= 1.34 \times 10^{-5} \times 10^{-3}$$

$$= \underline{1.34 \text{ T}} \quad \underline{\text{Ans}}$$

$$1. \frac{e^{-5}}{10^{-5}}$$



## Hall effect?

use of Hall effect;

(i) use to identify the type of semiconductor

(ii) to measure density of majority carrier.

(iii) to measure mobility of majority carriers (velocity per unit applied electric field)

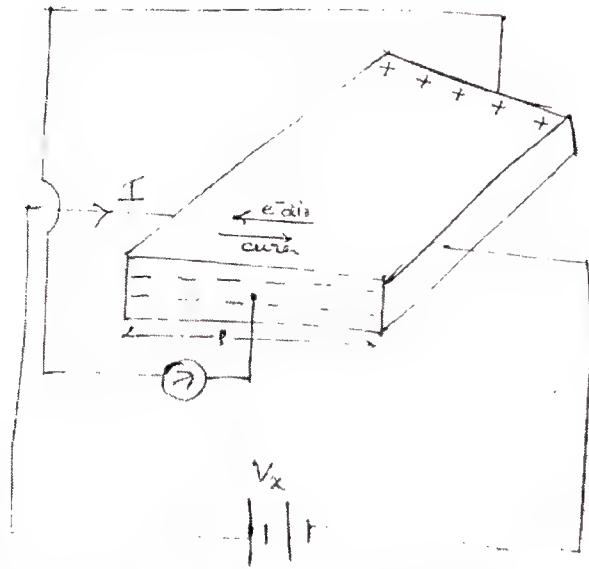
$$\vec{F} = I d\vec{l} \times \vec{B}$$

$$= \hat{x} \times \hat{z}$$

$$= -\hat{y}$$

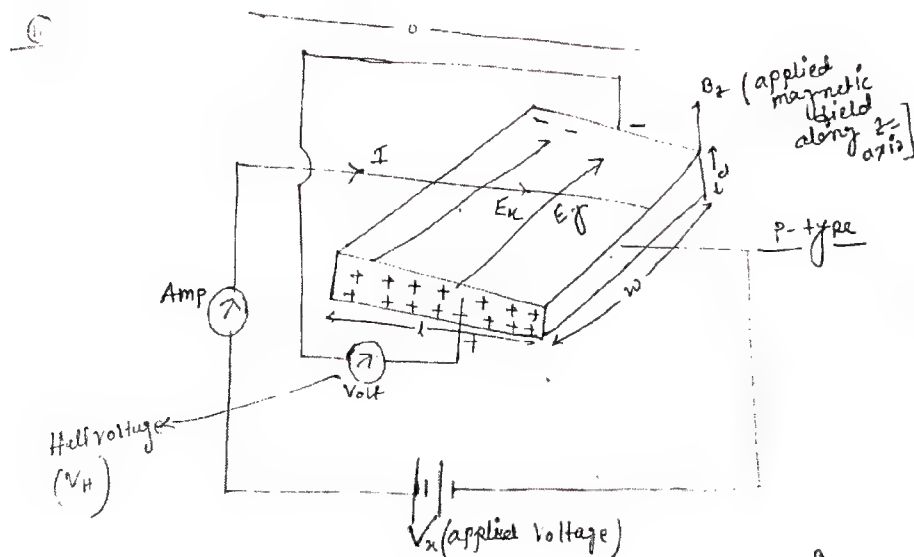
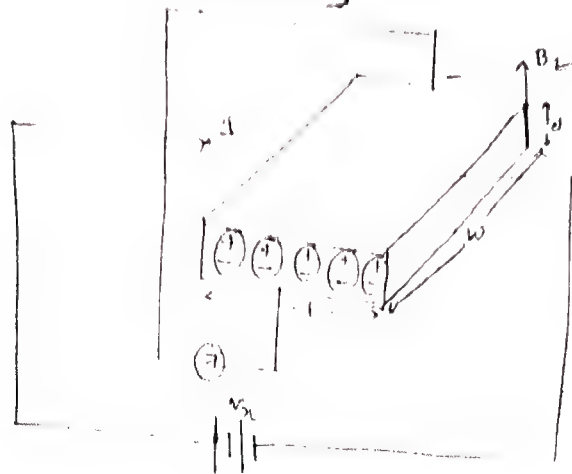
Hall voltage ( $V_H$ )

[ $V_H$  positive for p-type]

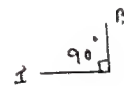


[ $V_H$  negative for n-type]

intrinsic number of  $e^-$  and number of hole are equal,  
 $V_H = 0$  [for intrinsic semiconductor]



Magnetic force on charge particle,



$$F = q \vec{v} \times \vec{B}$$

$$F_y = q v_x B_z \text{ (along negative y-direction)}$$

$E_y$  is the electric field created due Hall voltage  $V_H$ .

Electric force on charge particle =  $q E_y$  (along positive y-direction)  
 at equilibrium,

electric force on charge = magnetic force on charge

$$\rightarrow q v_x B_z = q E_y$$

$$v_x = \frac{E_y}{B_z} = \frac{V_H}{w B_z} \quad \text{--- (1)}$$

$$R_f = \frac{V_H}{I_H} \quad \left\{ \text{We consider } E_y = 0 \right\}$$

From ohm's law;

$$\text{Current density } (J) = \sigma E$$

$$J_x = J_{x1} = \sigma E_x \quad \text{--- (i)} \quad E_x = \text{Applied electric field,}$$

$$J_x = ne v_x \quad \text{--- (ii)}$$

$$\mu = \frac{v_x}{E_x} \quad (\text{velocity per unit electric field})$$

$$v_x = \mu E_x$$

$$\therefore J_x = ne \mu E_x$$

$$\Rightarrow \frac{J_x}{\text{Area}} = ne \mu E_x$$

$$\Rightarrow \frac{I_x}{wd} = ne v_x \quad \text{--- (iv)}$$

$$\Rightarrow v_x = \frac{I_x}{ne wd} \quad v_x = \text{drift velocity}$$

① and ④

$$\frac{I_H}{ne wd} = \frac{V_H}{B_z}$$

$$\Rightarrow \boxed{n = \frac{I_H B_z}{V_H e d}} \quad \begin{array}{l} n = \\ \text{number of} \\ \text{Carrier concentration} \\ \text{per unit volume} \end{array}$$

majority Carrier Concentration, for n-type and p-type.

$$\textcircled{iii} \text{ Mobility } \mu = \frac{v_x}{E_x}$$

from Equ<sup>n</sup> (iv),

$$\mu = \frac{I_x}{ne wd E_x}$$

$$\mu = \frac{I_x l}{ne wd V_x}$$

$$E_x = \frac{V_x}{l} \quad \text{applied Voltage}$$

This is expression to find that mobility.



$$\text{Hall Coefficient } (R_H) = \frac{1}{ne}$$

$$\text{unit} = \frac{\text{m}^3}{\text{C}} = \text{inverse unit of volume charge density}$$

$$\left\{ \begin{array}{l} \text{for p-type } R_H > 0; \\ \text{for n-type } R_H < 0; \end{array} \right\} e$$

Q. Find the Hall voltage if the applied magnetic field is 10 Tesla and width of semiconductor is 10 cm and drift velocity of charge carrier is  $3 \times 10^5 \text{ m/s}$

$$\left\{ \begin{array}{l} n = \frac{\text{charge}}{\text{volume}} \\ n = \frac{\text{C}}{\text{m}^3} \end{array} \right.$$

$$V_H = v_x \omega B_z$$

$$V_H = 10 \times 10 \times 10^{-2} \times 3 \times 10^5$$

$$V_H = 3 \times 10^5 \text{ V}$$

Q. Write down some use of Hall effect? [3rd]

(i)

(ii)

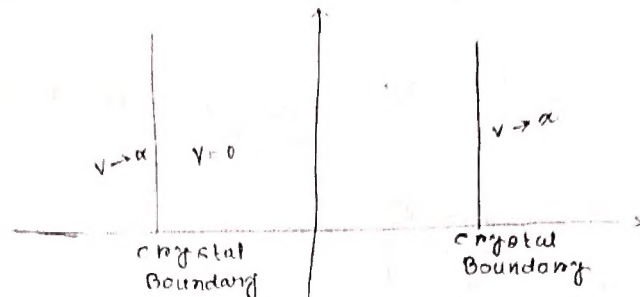
(iii)

$$R_H = \frac{E_H}{J_x B_z} = \frac{1}{en}$$

$$R_H = \frac{E_H}{J_x B_z}$$

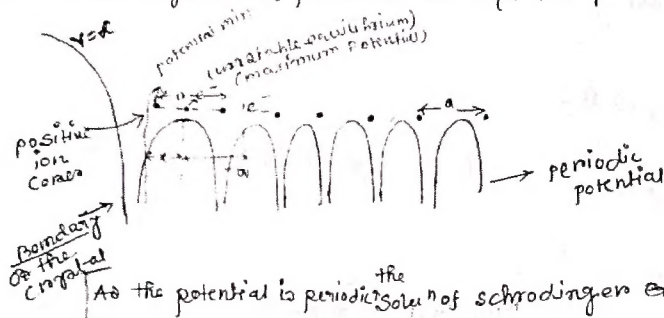
### Free electron theory

In free electron theory we assume that the potential inside the crystal is zero and potential is large at the boundary of the crystal.



But free electron theory cannot be able to explain why some metal are good conductor and electronic conductivity of semiconductor.

To explain this phenomenon, we assume that the potential inside the crystal is periodic as like its periodic structure.



As the potential is periodic, the solution of Schrodinger equation is also periodic.

\* Bloch find the solution of this periodic potential;

$$\Psi_k(x) = U_k(x) e^{ikx} \quad (\text{Bloch eqn}) \quad k = \text{Wave number}$$

As the solution is periodic;

$$\Psi_k(x+Na) = U_k(x+Na) e^{ik(x+Na)} \quad \begin{matrix} \text{total no. of lattice} \\ \text{point} \end{matrix}$$

$a = \text{spacing between lattice point}$

$$\begin{aligned} &= U_k(x) e^{ikx} \cdot e^{ikNa} \\ &= \Psi_k(x) \cdot e^{ikNa} \end{aligned} \quad \begin{matrix} U_k(x+Na) = U_k(x) \\ \text{so } U_k(x) \text{ must be periodic} \end{matrix}$$

$$\text{so, } e^{ikNa} = 1$$

$$e^{ikNa} = e^{i2n\pi}; \quad n = 1, 2, 3, 4$$

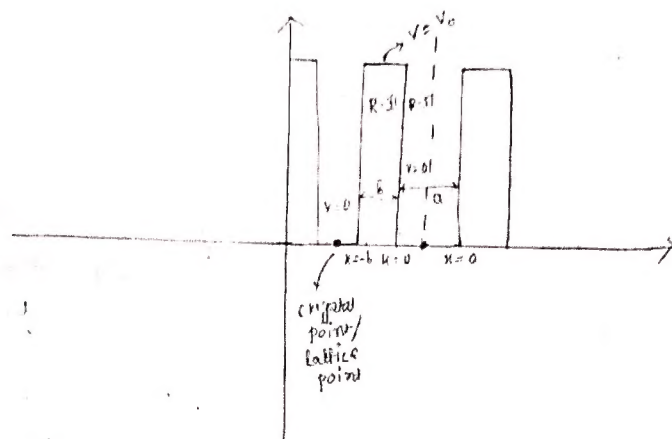
$$kNa = 2n\pi$$

$$\Rightarrow k = \frac{2n\pi}{Na}; \quad Na = L \quad (\text{length of crystal})$$

$$k = \frac{2n\pi}{L}$$

This is necessary condition to get the periodic solution.

Important:  
 The behaviour of energy of electron ( $E$ ) vs  $k$  value for the periodic potential inside the crystal can be explained by using Kronig-Penny model.  
 For simple of our calculation we consider the periodic potential as follows —



from Schrodinger's eqn;

$$-\frac{\hbar^2}{2m} \frac{\partial^2 \psi}{\partial x^2} + V\psi = E\psi; \quad R-I; \quad V=0$$

$$\Rightarrow -\frac{\hbar^2}{2m} \frac{\partial^2 \psi}{\partial x^2} = E\psi$$

$$\Rightarrow \frac{\partial^2 \psi}{\partial x^2} + \frac{2mE}{\hbar^2} \psi = 0 \quad \therefore \alpha^2 = \frac{2mE}{\hbar^2}$$

↓  
wave number

$$\Rightarrow \frac{\partial^2 \psi}{\partial x^2} + \alpha^2 \psi = 0 \quad \text{--- (I)}$$

$$m^2 + \alpha^2 = 0 \Rightarrow m^2 = -\alpha^2 \Rightarrow m = \pm i\alpha$$

$$\psi_1 = A e^{i\alpha x} + B e^{-i\alpha x} \quad \text{--- (II)}$$

for  $R-II$ ;

$$-\frac{\hbar^2}{2m} \frac{\partial^2 \psi}{\partial x^2} + V_0 \psi = E\psi \quad \text{--- (III)} \quad V = V_0$$

$$\Rightarrow \frac{\partial^2 \psi}{\partial x^2} + \frac{2mE}{\hbar^2} \psi = 0$$

$$\Rightarrow \frac{\partial^2 \psi}{\partial x^2} - \frac{2m}{\hbar^2} (V_0 - E) \psi = 0 \quad E < V_0$$

$$\therefore \beta^2 = \frac{2m}{\hbar^2} (V_0 - E)$$

$$\Rightarrow \frac{\partial^2 \psi}{\partial x^2} - \beta^2 \psi = 0$$

$$\psi_2 = C e^{\beta x} + D e^{-\beta x} \quad \text{--- (IV)}$$

to find the relation between A, B, C, D we have to use boundary conditions.

$$\psi_1(x)|_{x=-b} = \psi_2(x)|_{x=0} \quad \dots \quad \text{--- (V)}$$



$$\begin{vmatrix} 1 & 0 & 0 & 0 \\ i\alpha & i\alpha & \beta & \beta \\ i\alpha e^{i\alpha a} & i\alpha e^{i\alpha a} & -\beta e^{-\beta b} & -\beta e^{-\beta b} \\ i\alpha e^{i\alpha(a+b)} & i\alpha e^{i\alpha(a+b)} & -\beta e^{-\beta b} & -\beta e^{-\beta b} \end{vmatrix} = 0$$

on simplifying ;

$$\cos \alpha(a+b) = \left( \frac{\alpha^2 - \beta^2}{2\alpha\beta} \right) \sin \alpha a \sinh \beta b + \cos \alpha a \cosh \beta b$$

At low barrier,  $b \rightarrow 0$   $\sinh \beta b = \beta b$

$$\cosh \beta b \rightarrow 1$$

$$\cos \alpha a = \frac{\alpha^2 - \beta^2}{2\alpha\beta} \beta b \sin \alpha a + \cos \alpha a$$

$$\sinh \beta b = \frac{e^{\beta b} - e^{-\beta b}}{2} = \frac{1 + \beta b - (1 - \beta b)}{2}$$

$$\beta^2 - \alpha^2 = \frac{2m}{\hbar^2} (V_0 - E) = \frac{2mE}{\hbar^2}$$

$$= \frac{2m(V_0 - 2E)}{\hbar^2}$$

if  $V_0 \gg E$

$$\approx \frac{2mE}{\hbar^2} V_0$$

$$\cosh \beta b = \frac{e^{\beta b} + e^{-\beta b}}{2} = 1$$

From eqn (xiv)

$$\cos \alpha a = \frac{2mE V_0}{\hbar^2} \frac{1}{2\alpha\beta} \beta b \sin \alpha a + \cos \alpha a$$

$$= \frac{\alpha m b V_0}{\hbar^2} \frac{\sin \alpha a}{\alpha a} + \cos \alpha a$$

$$\cos \alpha a = P \frac{\sin \alpha a}{\alpha a} + \cos \alpha a$$

$$P = \frac{\alpha m b V_0}{\hbar^2} \quad (\text{penetration of potential})$$

This relation represents the relation between  $\hbar$  and  $E$  (energy electron)

$$\alpha = \sqrt{\frac{2mE}{\hbar^2}}$$